Model Examination

Grade: X **Mathematics** **Max. Marks:80**

Date: 01-12-2023 **Duration: 3Hrs**

Answers

1. **Option B**

The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Therefore, the distance between the points (2, -2) and (-1, x)

$= \sqrt{(-1-2)^2 + (x-(-2))^2}$

$\Rightarrow 5 = \sqrt{(-3)^2 + (x+2)^2}$

Squaring both sides, we get

$\Rightarrow (5)^2 = \left(\sqrt{9 + (x+2)^2}\right)^2$

$\Rightarrow 25 = 9 + (x+2)^2$

$\Rightarrow 25 = 9 + x^2 + 4 + 4x$

$\Rightarrow x^2 + 4x + 13 - 25 = 0$

$\Rightarrow x^2 + 4x - 12 = 0$

$\Rightarrow x^2 + 6x - 2x - 12 = 0$

$\Rightarrow x(x+6) - 2(x+6) = 0$

$\Rightarrow (x-2)(x+6) = 0$

$\Rightarrow x = 2, -6$

Hence, the values of x are 2 and -6.

2. **Option C**

Solution

It is given that, a = x^3y^2 and b = xy^3, where x, y are prime numbers.

LCM (a, b) = LCM (x^3y^2, xy^3)

= The highest of indices of x and y

= x^3y^3

Hence, LCM (a, b) is x^3y^3.

3. **Option b**
4. **Option a**
5. **Option c**

6. Option a
7. Option a
8. Option b
9. Option a
10. Option a
11. Option b
12. Option c 16
13. Option a
14. Option d
15. Option c
16. Option a
17. Option b
18. Option b
19. Option d
20. Option FF
21.

Given $\sqrt{3}$ is irrational number

Let $2 + \sqrt{3}$ is rational number.

then $2 + \sqrt{3} = \frac{p}{q}$

$\sqrt{3} = \frac{p - 2q}{q}$

Since $\sqrt{3}$ is irrational number but $\frac{p - 2q}{q}$ is rational it mean

irrational = rational

which contradict,

$\therefore$ our assumption is wrong and

$2 + \sqrt{3}$ is irrational number.

22.

Let the angle of elevation of the Sun is θ.

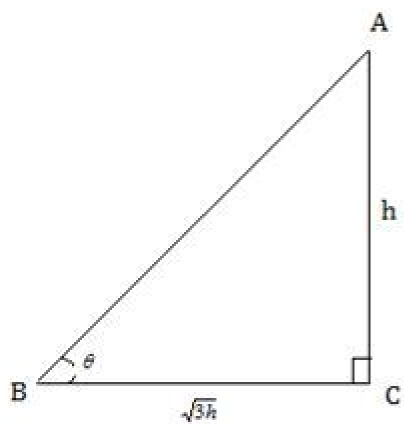
Given, height of pole = h

Now, in ΔABC,

$\tan\theta = \frac{AC}{BC} = \frac{h}{\sqrt{3}h}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30^\circ$

Hence, the angle of elevation of the Sun is 30°.



OR

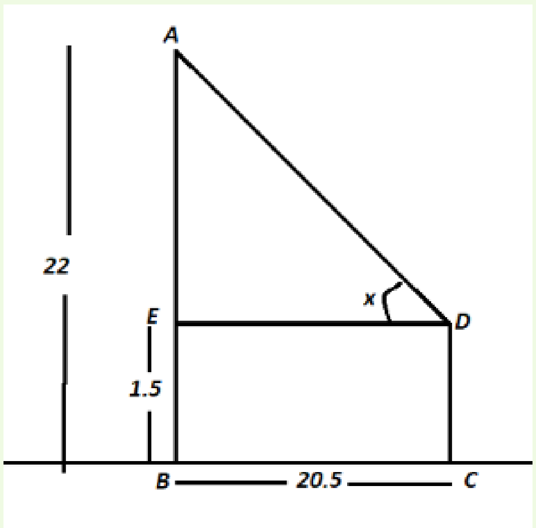
As the height of the Observer is 1.5

AE = 22 – 1.5 = 20.5

In $\triangle$AED

$$\tan\theta = \frac{AE}{ED} = \frac{20.5}{20.5} = 1 \Rightarrow \theta = 45^\circ$$

Therefore, answer is 45°



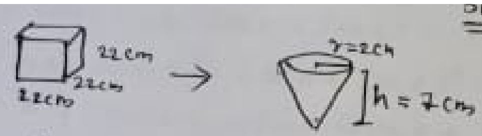
When three coins are tossed together, the total number of outcomes = 8
i.e., (HHH, HHT, HTH, THH, TTH, THT, HTT, TTT)

23.

23) When 3 coins are tossed together, the outcomes will be
{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
a) P[exactly 2 Heads] = $\frac{3}{8}$
b) P[atleast 2 Heads] = $\frac{4}{8} = \frac{1}{2}$
c) P[atmost 2 Heads] = $\frac{7}{8}$
d) P[exactly 1 Tail] = $\frac{3}{8}$

a) ⅜
b) 4/8=1/2
c) ⅞
d) 3/8

24.



Volume of ice-cream cube $= 22^3 = 22\times22\times22 = 10648\,cm^3$

volume of an ice cream cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\frac{22}{7}\times 2\times2\times7$

Let there be 'n' children.

Volume of n ice-cream cones = volume of ice-cube

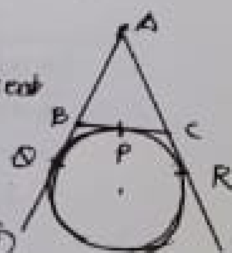
$n\times\frac{1}{3}\times\frac{22}{7}\times2\times2\times7 = 22\times22\times22$

$n = \frac{22\times22\times22\times3\times7}{22\times2\times2\times7} = \underline{363}$

$n = \underline{363}$

25.

Ans: Since the lengths of tangent from an external point to a circle are equal,



$BP = BQ$, $CP = CR$, $\longrightarrow$ (1)

$AQ = AR$ $\longrightarrow$ (2)

Perimeter of $\Delta ABC = AB + BC + AC$

$= AB + (BP + PC) + AC$

$= AB + BQ + CR + AC \longrightarrow$ by (1)

$= AQ + AR$

$= AQ + AQ$ [$\because AR = AQ$ by (2)]

Perimeter of $\Delta ABC = 2AQ$

$\therefore AQ = \frac{1}{2}$ (Perimeter of ΔABC)

26.

The timings at which the bells ring = 6m, 12m, 18m

L.C.M of 6, 12, 18 $= 2^2\times3^2$ $\qquad 6 = 2\times3$

$= 36$ $\qquad 12 = 2^2\times3$

$\qquad 18 = 2\times3^2$

$\therefore$ After 36 minutes the bells will ring together.

$\therefore$ The time after 6 a.m at which,

$\therefore$ The three bells ring together $= \underline{6:36}$ a.m

27.

1) P.T. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \cdot \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= (\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + \sec^2 A + 2\sin A \operatorname{cosec} A + 2\sec A \cdot \cos A$

$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2\cancel{\sin A} \cdot \frac{1}{\cancel{\sin A}} + 2 \times \frac{1}{\cancel{\cos A}} \cdot \cancel{\cos A}$

$= 1 + 1 + 1 + 2 + 2 + \cot^2 A + \tan^2 A$

$= 7 + \tan^2 A + \cot^2 A$

OR

P.T $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

L.H.S $= \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

$= \sqrt{\frac{1+\sin\theta}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1^2 - \sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} =$

$= \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{\cos\theta \cdot \cos\theta}}$

$= \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta$ = R.H.S

28.

Let OACBO be the minor sector.

$\angle AOB = 60°$

radius of the sector, $r = 15$ cm

Area of sector OACB $= \pi r^2 \frac{\theta}{360}$

$= \cancel{3.14}^{1.57} \times \cancel{15}^{5} \times 15 \times \frac{\cancel{60}}{\cancel{360}}$

$= 1.57 \times 5 \times 15$

$= \underline{117.75 \text{ cm}^2}$

In $\triangle OAB$, $\angle AOB = 60°$. Since $OA = OB = 15$ cm

$\Rightarrow \angle A = \angle B = x°$

$60 + x + x = 180$

$60 + 2x = 180$

$x = \frac{180 - 60}{2} = \frac{120}{2} = 60°$

$\Rightarrow \triangle OAB$ is equilateral $\triangle$.

Area of $\triangle OAB = \frac{\sqrt{3}}{4} \times a^2$

$= \frac{\sqrt{3}}{4} \times 15 \times 15 = \frac{1.73 \times 15 \times 15}{4}$

$= \underline{97.3125 \text{ cm}^2}$

Area of minor segment = Area of minor sector − Area of $\triangle OAB$

$= 117.75 \text{ cm}^2 - 97.3125 \text{ cm}^2$

$= \underline{20.4375 \text{ cm}^2}$

Area of major segment $= \pi r^2$ − Area of minor segment

$= 3.14 \times 15 \times 15 - 20.4375$

$= 706.5 - 20.4375$

$= \underline{686.0625 \text{ cm}^2}$

OR

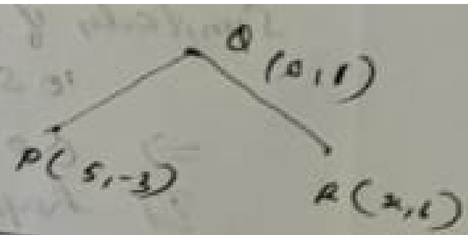
Here $\theta = 120°$, $r = 25$ cm

Total area cleaned by 2 wiper $= \pi r^2 \cdot \frac{\theta}{360} \times 2$

$= 2 \times 3.14 \times 25 \times 25 \times \frac{\cancel{120}}{\cancel{360}_{3}}$

$= \underline{1308.33 \text{ cm}^2}$

29.



$$PQ^2 = QR^2$$

$$\sqrt{(5-0)^2+(-3-1)^2} = \sqrt{(x-0)^2+(6-1)^2}$$

$$5^2+(-4)^2 = x^2+5^2$$

$$\cancel{25}+16 = x^2+5^2$$

$$x^2 = 16 \qquad x^2+\cancel{25}$$

$$\Rightarrow x = \pm 4$$

Let $R(4,6)$ an (6)

Distance of $QR = \sqrt{(4-0)^2+(6-1)^2}$

$$= \sqrt{4^2+5^2} = \sqrt{16+25} = \underline{\sqrt{41}} \text{ units}$$

$$PQ = \sqrt{(4-5)^2+(6--3)^2} = \sqrt{(-1)^2+9^2}$$

$$= \sqrt{1+81} = \underline{\sqrt{82}} \text{ units}$$

30.

$d = 480$ km

$s = x$

Let the normal speed of the train be 'x' km/hr

Time taken to travel 480 km $t_1 = \frac{d}{s} = \frac{480}{x}$

If the speed is 8 km/hr less it becomes $x - 8$,

then time taken, $t_2 = \frac{480}{x-8}$

As per the question,

$$t_2 = t_1 + 3$$

$$\frac{480}{x-8} = \frac{480}{x} + 3$$

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$480\left[\frac{1}{x-8} - \frac{1}{x}\right] = 3$$

$$480\left[\frac{x-(x-8)}{x(x-8)}\right] = 3$$

$$480\left[\frac{\cancel{x}-\cancel{x}+8}{x^2-8x}\right] = 3$$

$$\frac{8}{x^2-8x} = \frac{1}{160}$$

$$x^2 - 8x - 1280 = 0$$

$$x^2 - 40x + 32x - 1280 = 0$$

$$x(x-40) + 32(x-40) = 0$$

$$(x-40)(x+32) = 0$$

$$x = 40, -32$$

We neglect $x = -32$, as speed can't be negative

$\therefore x = 40$

$\therefore$ Speed of the train $= 40$ km/h

31.

Take a point Q other than P on the tangent XY. Clearly Q lies outside the circle.

In the figure, $OP = OR = r$

Now $OQ = OR + RQ$

$\therefore OQ > OP$

Similarly if S is any other point on XY. Then,

$OS > OP$ by the same argument

$\Rightarrow OP$ is less than OS, and OQ...

It happens for every point on XY, other than the point P.

But of all the points on XY, perpendicular is the short distance from a point to the line XY

$\Rightarrow OP \perp XY$

Tangent at any point is $\perp$r to the radius.

32.

32) Required amount $= Rs\ 2500$, $n = 12$

The terms are, 100, 120, 140, ...

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$= \frac{12}{2}[2(100)+(12-1)20]$$
$$= 6[200+11(20)]$$
$$= 6[200+220]$$
$$= 6 \times 420$$
$$= \underline{\underline{2520}}$$

She will have Rs 2520 after 12 weeks

Yes, She can send her daughter to school.

OR

Let the first 3 terms be $a-d$, a, $a+d$

$$a-\cancel{d}+a+a+\cancel{d} = 33$$
$$3a = 33$$
$$a = \frac{33}{3} = 11$$
$$a = \underline{\underline{11}}$$

$$(a-d)(a+d) = a+29$$
$$a^2-d^2 = 11+29$$
$$11^2-d^2 = 40$$
$$121-d^2 = 40$$
$$d^2 = 121-40 = 81$$
$$d = \underline{\underline{\pm 9}}$$

if $d=9$, when $a=11$, $a-d = 11-9 = 2$

$$a = 11$$
$$a+d = 11+9 = 20$$

$\therefore$ The terms are $\underline{\underline{2, 11, 20}}$

If $a=11$, $d=-9$

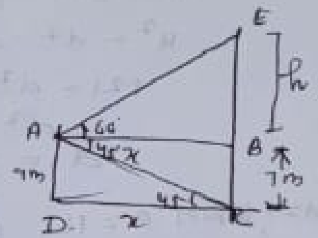
then $a-d = 11-(-9) = 20$

$$a = 11$$
$$a+d = 11+(-9) = 2$$

$\therefore$ The terms are 2, 11, 20 OR 20, 11, 2

33. Proof of Basic Proportionality theorem

34.



In ΔABE

$$\tan 60 = \frac{BE}{AB}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3}x \longrightarrow (1)$$

In right ΔABC

$$\tan 45° = \frac{AD}{DC}$$

$$1 = \frac{7}{x}$$

$$x = 7$$

putting $x = 7$ in (1)

$$h = \sqrt{3} \times 7 = 7\sqrt{3}\ m$$

$\therefore$ Height of the tower $= 7 + h$

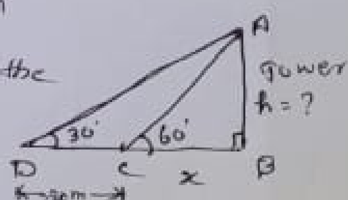
$$= 7 + 7\sqrt{3}$$

$$= 7(\sqrt{3}+1)\ m$$

OR

ii) Let AB be the tower and BC be the canal. C be the point on the other side of the canal.

Let $AB = h$, $BC = x$



In ΔBAC, $\tan 60 = \frac{h}{x}$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \longrightarrow (1)$$

In ΔABD;

$$\tan 30 = \frac{AB}{x+20}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+20}$$

$$\sqrt{3}h = x + 20$$

$$\sqrt{3} \times \sqrt{3}x = x + 20$$

$$3x = x + 20$$

$$2x = 20$$

$$x = \frac{20}{2} = 10$$

put $x = 10$ in (1)

$$h = \sqrt{3}x = \sqrt{3} \times 10 = 10\sqrt{3}\ m$$

width of the canal $= 10m$

Height of the tower $= h = 10\sqrt{3}\ m$

35.

Here, it is given that Median = 28.5 and n = $\sum f_i = 60$

Cummulative frequency table for the following data is given.

Here n = 60 $\Rightarrow \frac{n}{2} = 30$

Since, median is 28.5, median class is 20 – 30

Hence, l = 20, h = 10, f = 20, c.f. = 5 + x

Therefore, Median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right)h$

$28.5 = 20 + \left(\frac{30 - 5 - x}{20}\right)10$

$\Rightarrow 28.5 = 20 + \frac{25 - x}{2}$

$\Rightarrow 8.5 \times 2 = 25 - x$

$\Rightarrow x = 8$

Also, 45 + x + y = 60

$\Rightarrow y = 60 - 45 - x = 15 - 8 = 7.$

Hence, x = 8, y = 7

Class - interval	Frequency	Cumulative frequency
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$
Total	n = 60	

OR

nber of Wickets	Number of Bowlers (f_i)	Class Mark (x_i)	$f_i x_i$
20 – 60	7	$\frac{20+60}{2} = 40$	$7 \times 40 = 280$
60 – 100	5	$\frac{60+100}{2} = 80$	$5 \times 80 = 400$
100 – 150	16	$\frac{100+150}{2} = 125$	$16 \times 125 = 2000$
150 – 250	12	$\frac{150+250}{2} = 200$	$12 \times 200 = 2400$
250 – 350	2	$\frac{250+350}{2} = 300$	$2 \times 300 = 600$
350 – 450	3	$\frac{350+450}{2} = 400$	$3 \times 400 = 1200$
	$\sum f_i = 45$		$\sum f_i x_i = 6880$

This problem can be solved using direct method as class interval is different.

We know tha,

$$\text{Mean } (\overline{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

=152.89

36.

1. To find the angle of elevation, $\tan\ \theta = \frac{\text{height of the tower}}{\text{dis tan ce from the tower}} = \frac{42}{42} = 1$

$$\sqrt{3} = \frac{42}{\text{dis tan ce}}$$

$$\text{dis tan ce} = \frac{42}{\sqrt{3}}$$

$$\text{dis tan ce} = 24.64\ \text{m}$$

$\theta = \tan^{-1}(1) = 45°$

2. To find the distance, $\tan\ 60° = \frac{\text{height of the tower}}{\text{dis tan ce}} = \frac{42}{\text{dis tan ce}}$

$$\sqrt{3} = \frac{42}{\text{dis tan ce}}$$

$$\text{dis tan ce} = \frac{42}{\sqrt{3}}$$

$$\text{dis tan ce} = 24.64\ \text{m}$$

3. To find the height of the verticle tower, $\tan\ 60° = \frac{\text{height of the tower}}{\text{dis tan ce}}$

$$\sqrt{3} = \frac{\text{height of the tower}}{20}$$

$$\text{height of the tower} = 20\sqrt{3}$$

4. To find the angle of elevation of the sun, $\tan\ \theta = \frac{\text{height of the tower}}{\text{dis tan ce from the tower}}$

$$= \frac{1}{1} (\text{sin ce the ratios are in } 1:1)$$

$$\theta = \tan^{-1}(1) = 45°$$

37.

Volume of cylindrical container=πR2

H=π×62×15=540π㎤

Let radius of base of conical portion be 'r', then diameter=2r

and height=2×2r=4r

10×volume of each cone=volume of container

⇒10[3πr2h+32πr3]=540π⇒10×31π[r2×4r+2r3]=540π

⇒6r3=162⇒

r3=27⇒r=3

27=3cm

∴Diameter =2r=2×3=6cm

38.

Given that

First row has 30 seats, and each succeeding row has 10 seats more than the previous one

So, this is **an AP** with

First term = **a = 30**

Common difference = **d = 10**

We need to find how many seats will be there in the 10^{th} row

i.e. We need to **find a_{10}**

We know that

$a_n = a + (n - 1)d$

Putting n = 10, a = 30, d = 10

$a_{10} = \mathbf{30 + (10 - 1) \times 10}$

$= 30 + 9 \times 10$

$= 30 + 90$

$= 120$

Thus, there are **120 seats** in the 10^{th} row